Quadratics1a - Quadratic Functions: $f(x) = ax^2 + c$

Standards: **F-IF.4** & **F-IF.7**

GLOs: #1 Self Directed Learner

Math Practice: Look for and make use of structure

Learning Target:
What are the characteristics of a Quadratic graph?

(Review)

(erase to show)

Linear Functions can be symbolically represented as **f(x) = mx + b** where m & b are numbers and x is an independent variable.

When we say "Numbers" they may not necessarily be "nice" numbers. They just have to be **Real Numbers** .

#s that can plotted on a # line.

Definition: a **Quadratic Function** is a function that can be symbolically represented as $f(x) = ax^2 + bx + c$ where a, b, & c are numbers, x is an independent variable, and the co-efficient "a" of cannot equal zero.

This is referred to as the ***Standard Form*** of a Quadratic Equation.

Reflection: Why do you think we require that "a" cannot equal 0? If a = 0, then it becomes a linear function.

$f(x) = ax^2 + bx + c$

$f(x) = \underbrace{0x^2}_{0} + bx + c \longrightarrow f(x) = bx + c$

For each function below indicate if it represents a quadratic function or not.

If it is not, explain why not. $f(x) = ax^2 + bx + c$

Is this a Quadratic Function?	YES	NO	
$f(x) = -2x^2 + 6x + 9$	✓		
$f(x) = 0x^2 - x + 12$		✓	a=0
$f(x) = \frac{-23}{17}x^2 - 11x + 0$	✓		
$f(x) = x^2 + 15$ $x^2+0x+15$	✓		
$f(x) = -x^2 + \frac{7}{11}x - 4$	✓		
$f(x) = 6$ $0x^2+0x+6$		✓	a=0
$f(x) = 9 + 6x - 25x^2$ $f(x) = -25x^2+6x+9$	✓		
$f(x) = 2x - 3$		✓	a=0

The simplest quadratic function, the one we refer to as the **"Parent Function"** for quadratic functions, is the function defined as $f(x) = x^2$

We begin our investigation of quadratic functions with the parent function.

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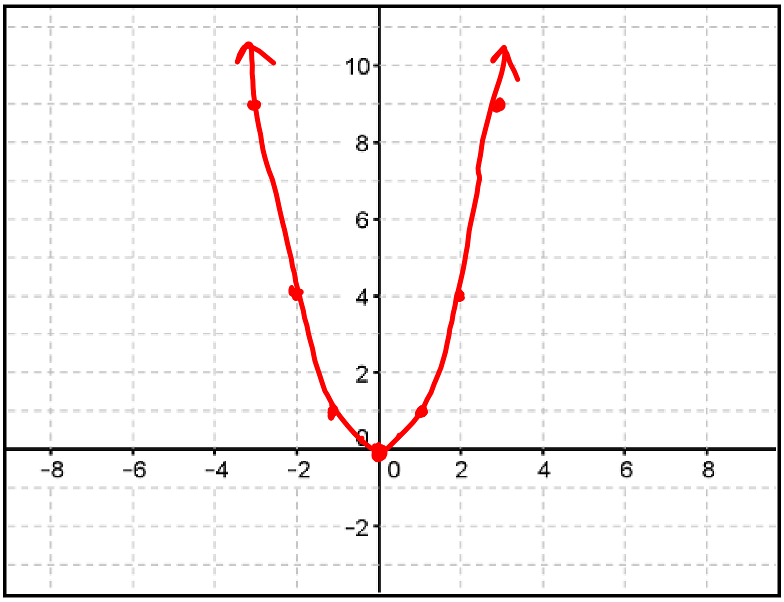
1) Fill in the following tables of values for $f(x)=x^2$ and use your results to graph $f(x)=x^2$.

x	$f(x)=x^2$
0	0
1	1
2	4
3	9
4	16

x	$f(x)=x^2$
-1	1
-2	4
-3	9
-4	16

x	$f(x)=x^2$
1/2	1/4
1/3	1/9
1/4	1/16



(erase to show)

The graph of $f(x)=x^2$ is called a **Parabola**

(U-shaped graph).

$f(x)=x^2$ is what we call an **even function** because the graph has the same positive heights on both the left and right side of the y-axis making the y-axis a **line of symmetry** In other words, the height at any negative x-value is the same as the height at the corresponding positive x-value. The lowest point is called the **vertex**.

Notes: We will show in a later lesson that the graph of every quadratic function is a parabola.
You may assume this fact as you proceed.

(erase to show)

The **domain** (set of inputs) for the parent function is the set of real numbers, since is defined for all numbers. The domain for every quadratic function is the set of real numbers.

The **Range** (set of outputs) is restricted to all non-negative real numbers since squaring an input never results in a negative number. Another way of thinking of this property is that for each nonnegative number there is a point on the graph of with that height. In fact, for each positive number there are two points on the graph with that height. The range will vary for different quadratic functions.

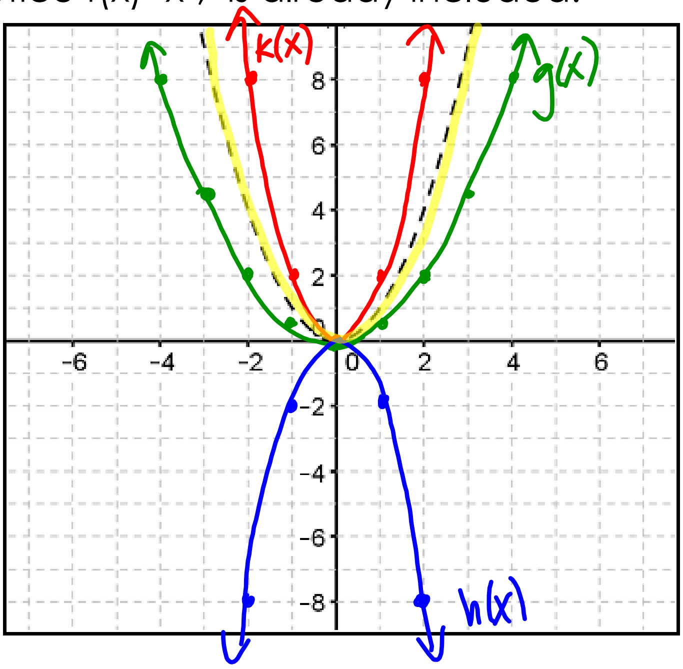
2) Fill in the following tables of values for functions of the form $f(x) = ax^2$ for various values of the leading coefficient "a."

x	$k(x) = 2x^2$
0	0
1	2
2	8
3	18
4	32

x	$g(x) = \frac{1}{2}x^2$
0	0
1	½
2	2
3	4½
4	8

x	$h(x) = -2x^2$
0	0
1	-2
2	-8
3	-18
4	-32

3) Graph k, g, and h above on the same coordinate axes below. Notice f(x)=x², is already included.



4) Reflections: Describe the effect that different values of "a" have on the graph in comparison to that of the parent function f(x)=x².

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If $a > 1$ then the graph will be narrower/skinnier than the parent function.

If $0 < a < 1$ then the graph will be wider/fatter than the parent function.

concave up

If $a < 0$ then the graph will be concave down (open down)

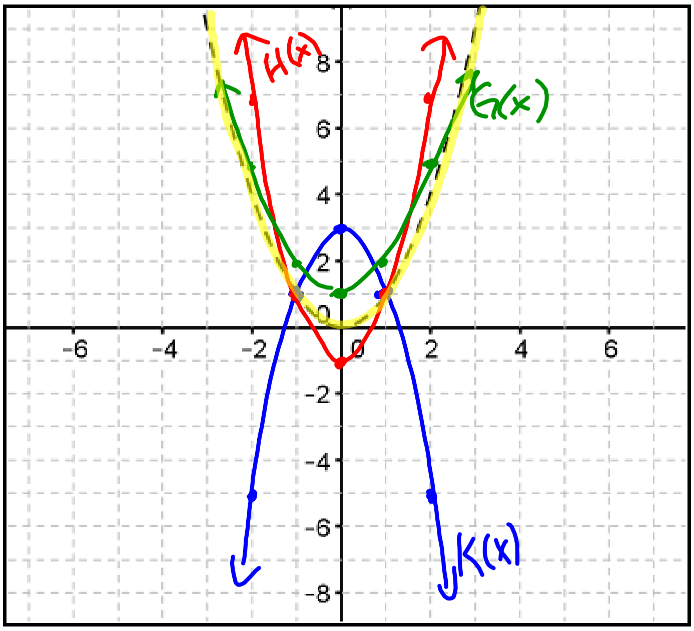
5) Fill in the following tables of values for functions of the form $f(x) = ax^2 + c$ for various values of the leading coefficient "a" and the constant term "c."

x	$G(x) = x^2 + 1$
0	1
1	2
2	5
3	10
4	17

x	$H(x) = 2x^2 - 1$
0	-1
1	1
2	7
3	17
4	31

x	$K(x) = -2x^2 + 3$
0	3
1	1
2	-5
3	-15
4	-29

6) Graph G, H, & K above on the same coordinate axes below. Notice f(x)=x2, is already included.



7) Reflections: What is the role of the constant term "c" in the quadratic function $f(x) = ax^2 + c$ in terms of its effect on the graph of the quadratic function defined by $g(x) = ax^2$?

The c- value shifts the parabola c units vertically.

Making a table of values is one way to graph a quadratic function, but in most instances it is much easier to simply identify the effect that symbolic changes have on the parent function.

(in class)

Summary of the effects different values of a and b have on the graph of $g(x) = ax^2 + c$:

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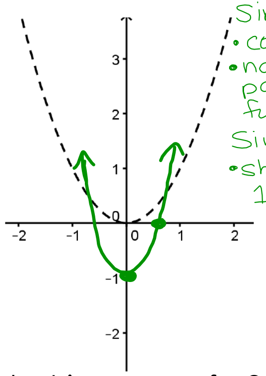
- Changing the value of "*a*" in $g(x) = ax^2$ simply increases all **heights** on the parent function defined by $f(x) = x^2$ by a factor of "*a*", provided "*a*" is positive. Thus, $g(x) = 3x^2$ will appear to grow faster than the parent function and $h(x) = \frac{1}{2}x^2$ will appear to grow slower than the parent function. The vertex remains the same.

- **Negative** values of "*a*" will reflect the parent function across the x-axis, in which case it will be **concave down** instead of concave up. The vertex remains the same, but it is now a maximum instead of a minimum.

- Adding a non-zero number "c" to $g(x) = ax^2$ will shift the function **up** "c" units if "c" is **positive** and shift it **down** if "c" is **negative** The x-value of the vertex remains zero, but the *y*-value of the vertex is shifted up or down "c" units.

8) Quickly sketch graphs of the following quadratic functions. You have 10 minutes for this exercise, so concentrate only on the most important aspects of the graph in comparison to the parent function. The graph of the parent function $f(x) = x^2$ is already given.

a) $g(x) = 2x^2 - 1$



Since $a=2$
- concave up
- narrower than parent function

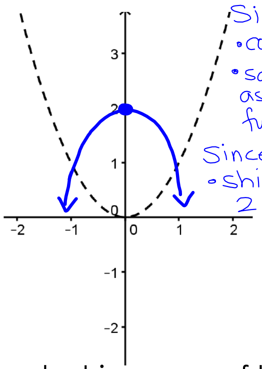
Since $c=-1$
- shift down 1 unit

what is range of g?

$y \geq -1$

$g(x) \geq -1$

b) $h(x) = -x^2 + 2$



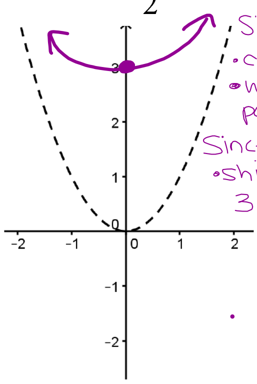
Since $a=-1$
- concave down
- same width as parent function

Since $c=2$
- shift up 2 units

what is range of h?

$h(x) \leq 2$

c) $m(x) = \frac{1}{2}x^2 + 3$



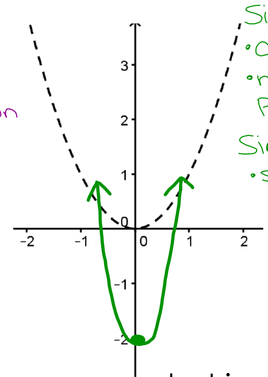
Since $a=\frac{1}{2}$
- concave up
- wider than parent function

Since $c=3$
- shift up 3 units

what is range of m?

$m(x) \geq 3$

d) $n(x) = 3x^2 - 2$



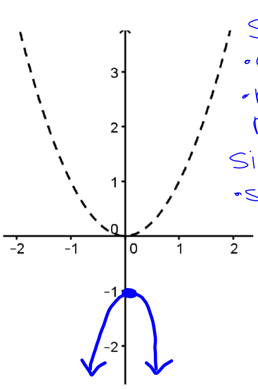
Since $a=3$
- concave up
- narrower than parent function

Since $c=-2$
- shifts down 2 units

what is range of n?

$n(x) \geq -2$

e) $k(x) = -2x^2 - 1$



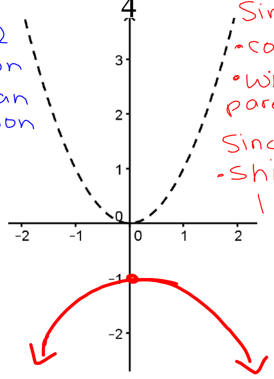
Since $a=-2$
- concave down
- narrower than parent function

Since $c=-1$
- shifts down 1 unit

what is range of k?

$k(x) \leq -1$

f) $p(x) = -\frac{1}{4}x^2 - 1$



Since $a=-\frac{1}{4}$
- concave down
- wider than parent function

Since $c=-1$
- shifts down 1 unit

what is range of p?

$p(x) \leq -1$